Magnetism

Unit 6

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Contents

6.1 Concepts of a magnetic field

By the end of this section you should be able to:

- Define magnetic field.
- State the properties of magnetic field lines.
- Describe the properties, including the three-dimensional nature, of magnetic fields.

Magnetic fields

You learnt about magnets and **magnetic fields** in Grade 10. You know that a magnet has two poles, which we label 'north' and 'south' (see Figure 6.1).

Figure 6.1

A magnetic field is a region where a magnet exerts a force. **Magnetic field lines** (also called magnetic flux lines), which show where the magnetic field has the same strength, point from the north pole of a magnet to the south pole of the magnet, as shown in Figure 6.2.

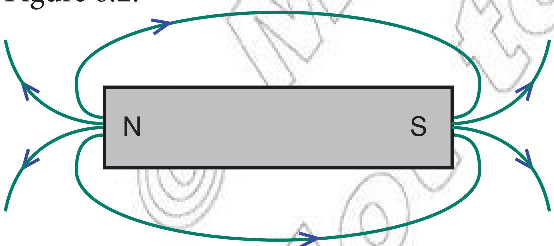

Figure 6.2

A magnetic field is a vector quantity as it has both magnitude (which depends on the strength of the magnet which produced it) and direction. The stronger the magnetic field, the closer the flux lines. Using this idea, we sometimes refer to the magnetic field strength as the magnetic flux density. The symbol for **magnetic flux density** is B and the unit of measurement is the tesla (T).

KEY WORDS

magnetic field *a region where a magnet exerts a force*

magnetic field lines *show where the magnetic field has the same strength*

magnetic flux density *a measure of the strength of the magnetic field – shown visually by how close the lines of flux are to each other*

Activity 6.1: Demonstrating the magnetic flux lines around a magnet

In a small group, devise a method for showing the magnetic flux lines around a bar magnet

a) in two dimensions

b) in three dimensions.

(Hint: Think about how you could use a clear plastic bottle, liquid glycerine, iron filings and a bar magnet to show the field in three dimensions.)

Repeat this activity for a different magnet (e.g. a horseshoe magnet) of your choice.

The strength of a magnetic field is also indicated by the quantity of flux (Φ) through any given area. Flux is measured in Webers (Wb). To find the flux for a particular region you multiply the area of the region by the component of flux density perpendicular to the area:

$\Phi = B\sin\theta \times A$

Worked example 6.1

The bar magnet in Figure 6.3 causes a magnetic field with a strength of 30 mT at an angle of 75° to the region of area A, how much flux will be contained by this region if the area is 5 cm^2?

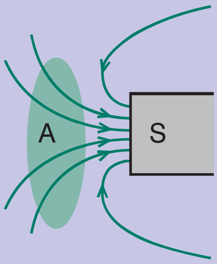

Figure 6.3

Φ (Wb)	B (T)	sin θ	A (m^2)
?	30×10^{-3}	0.966	5×10^{-4}

Use $\Phi = B\sin\theta \times A$

$= 30 \times 10^{-3} \times 0.966 \times 5 \times 10^{-4}$

$= 1.45 \times 10^{-5}$ Wb

Summary

In this section you have learnt that:

- A magnetic field is a region where a magnet exerts a force. Magnetic field lines, (also called magnetic flux lines) which show where the magnetic field has the same strength, point from the north pole of a magnet to the south pole of the magnet.
- A magnetic field is a vector quantity as it has both magnitude (which depends on the strength of the magnet which produced it) and direction. The stronger the magnetic field, the closer the flux lines. Using this idea, we sometimes refer to the magnetic field strength as the magnetic flux density. The symbol for magnetic flux density is B and the unit of measurement is the tesla (T).
- The strength of a magnetic field is also indicated by the quantity of flux (Φ) through any given area. Flux is measured in Webers (Wb). To find the flux for a particular region you multiply the area of the region by the component of flux density perpendicular to the area:
  $\Phi = B\sin\theta \times A$

Review questions

1. What is a magnetic field?
2. Copy the diagrams and draw in the magnetic flux lines for each magnet.

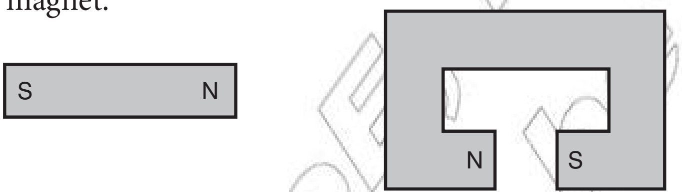

Figure 6.4

3. How could you demonstrate the three-dimensional nature of the magnetic field around a bar magnet?
4. The bar magnet in Figure 6.5 causes a magnetic field with a strength of 20 mT at an angle of 60° to the region of area A, how much flux will be contained by this region if the area is 10 cm^2?

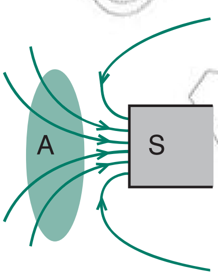

Figure 6.5

KEY WORDS

magnetism *describes how the atoms of materials respond to a magnetic field*

diamagnetism *the tendency of a material to oppose an applied magnetic field*

paramagnetic *materials have unpaired electrons, which will tend to align themselves in the same direction as the applied magnetic field, thus reinforcing it*

ferromagnetic *materials have unpaired electrons, which will align with the applied magnetic field and parallel to each other. They keep this alignment even when the applied field is removed.*

lodestone *a naturally magnetised piece of the mineral magnetite*

DID YOU KNOW?

A **lodestone** or **loadstone** is a naturally magnetised piece of the mineral magnetite. They are naturally occurring magnets that attract pieces of iron. Ancient people first discovered the property of magnetism in lodestone. The earliest written reference to magnetism occurs in a book from the 4th Century BCE in ancient China. Pieces of lodestone, suspended so they could turn, were the first magnetic compasses, and their importance to early navigation is indicated by the name lodestone, which in Middle English means 'course stone' or 'leading stone'.

6.2 The Earth and magnetic fields

By the end of this section you should be able to:

- Describe magnetic properties of matter.
- Distinguish between the terms diamagnetic, paramagnetic and ferromagnetic materials.
- Describe the causes of the Earth's magnetism.

Magnetic properties of matter

We use the term **magnetism** to describe how the atoms of materials respond to a magnetic field. **Diamagnetism** is a property of all materials. It is the tendency of a material to oppose an applied magnetic field. Some materials will, however, reinforce a magnetic field because they have unpaired electrons, which will tend to align themselves in the same direction as the applied magnetic field, as shown in Figure 6.6. These are known as **paramagnetic** materials.

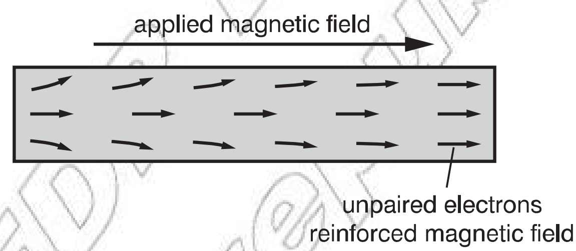

Figure 6.6 *The alignment of unpaired electrons*

Ferromagnetic materials have unpaired electrons. In addition to the tendency of these electrons to align themselves in the same direction as an applied magnetic field, they will also align themselves so that they are parallel to each other. This means that, even when the applied field is removed, the electrons in the material maintain a parallel orientation. Examples of ferromagnetic materials are nickel, iron, cobalt and their alloys.

The causes of the Earth's magnetism

The Earth can be thought of as a huge magnet. The geographic north pole is the south pole of the magnet, and the geographic south pole is the north pole of the magnet. Hence, the needle on a compass will be attracted to the geographic north pole (see diagram).

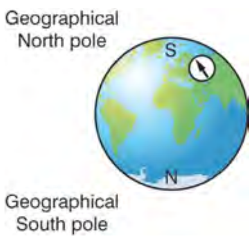

Figure 6.7 *The poles of the Earth's magnetic field*

There is no simple answer to the question 'where does the Earth's magnetism come from?'. In Grade 10, you learnt that magnetic fields surround electric currents. We can form a theory that circulating electric currents in the molten iron core of the Earth produce the magnetic field. We do not know how this 'dynamo effect' works in detail, but the rotation of the Earth plays a part in generating the currents that are presumed to be the source of the magnetic field. You will discuss the horizontal and vertical components of the Earth's magnetic field in more detail in section 6.6.

DID YOU KNOW?

The properties of magnets and the dry compass were discovered in 1282 by a Yemeni physicist, astronomer and geographer, Al-Ashraf.

Activity 6.2: Plotting the combined magnetic field of the Earth and a bar magnet

Place a bar magnet with its north pole facing geographic south on large sheet of paper. Use a compass to plot the combined magnetic field of the Earth and this magnet. Find any neutral points (that is, points where there is no field).

Summary

In this section you have learnt that:

- The term **magnetism** describes how the atoms of materials respond to a magnetic field.
- **Diamagnetism** is a property of all materials. It is the tendency of a material to oppose an applied magnetic field.
- **Paramagnetic** materials reinforce a magnetic field because they have unpaired electrons which will tend to align themselves in the same direction as the applied magnetic field.
- **Ferromagnetic** materials have unpaired electrons. In addition to the tendency of these electrons to align themselves in the same direction as an applied magnetic field, they will also align themselves so that they are parallel to each other. This means that, even when the applied field is removed, the electrons in the material maintain a parallel orientation.
- We can form a theory that circulating electric currents in the molten iron core of the Earth produce the magnetic field. We do not know how this 'dynamo effect' works in detail, but the rotation of the Earth plays a part in generating the currents which are presumed to be the source of the magnetic field.

Review questions

1. Explain the difference between diamagnetic, paramagnetic and ferromagnetic materials.
2. Outline a theory that can explain the Earth's magnetic field.

6.3 Motion of charged particles in a magnetic field

By the end of this section you should be able to:

- Describe the motion of a charged particle in a magnetic field.
- Identify a moving charge sets up a magnetic field.
- Use the equation $F = qv \times B$ to determine the magnitude and direction of the force.
- Use the expression for the force on a charged particle in a magnetic field.
- Solve problems on the motion of charged particles in electric and magnetic fields.
- Describe the path if $\theta \neq 90°$.
- Describe J.J. Thompson's experiment of charge to mass ratio.
- Determine the value of charge mass ratio for this specific experiment.

The motion of a charged particle in a magnetic field

A moving charged particle creates a magnetic field. A charged particle moving in a magnetic field will create a force, $\vec{F}$. The magnitude of this force depends on:

- the speed of the particle, $\vec{v}$
- the strength of the magnetic field, $\vec{B}$.

The force can be calculated using the vector cross product

$$\vec{F} = q(\vec{v} \times \vec{B})$$

From the definition of the vector cross product, we can say that the magnitude of the force is

$$F = qvB\sin\theta$$

where θ is the angle between $\vec{v}$ and $\vec{B}$. We can find the direction of the force by using the right hand rule, as shown in Figure 6.8.

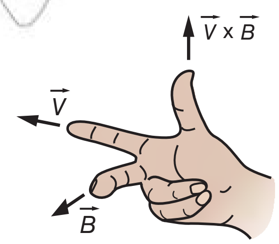

***Figure 6.8** The right hand rule for magnetic force*

Worked example 6.2

a) Find the size of the force felt by an electron travelling perpendicular to the Earth's magnetic field at 500 m/s. (The charge on an electron is 1.6×10^{-19} C and the magnitude of the Earth's magnetic field is 5×10^{-5} T.)

b) In what direction will the force act?

a)

F (N)	*q* (C)	*v* (m/s)	*B* (T)	sin θ
?	1.6×10^{-19}	500	5×10^{-5}	1

Use

$\vec{F} = q(\vec{v} \times \vec{B})$

$F = qvB\sin\theta$

$= 1.6 \times 10^{-19} \times 500 \times 5 \times 10^{-5} \times 1$

$= 4 \times 10^{-21}$ N

b) The force will act in a direction that is perpendicular to both the Earth's magnetic field and the direction in which the electron is travelling.

Worked example 6.3

Find the size of the force felt by an electron travelling at an angle of 30° to the Earth's magnetic field at 500 m/s. (The charge on an electron is 1.6×10^{-19} C and the magnitude of the Earth's magnetic field is 5×10^{-5} T.)

F (N)	*q* (C)	*v* (m/s)	*B* (T)	sin θ
?	1.6×10^{-19}	500	5×10^{-5}	0.5

Use

$\vec{F} = q(\vec{v} \times \vec{B})$

$F = qvB\sin\theta$

$= 1.6 \times 10^{-19} \times 500 \times 5 \times 10^{-5} \times 0.5$

$= 2 \times 10^{-21}$ N

Activity 6.3: Determining the strength of a magnetic field

In a small group, discuss how you could determine the strength of a magnetic field if you had a small test charge whose acceleration you were able to measure.
(Hint: start with Newton's second law.)

The motion of charged particles in electric and magnetic fields

Particles can move in both magnetic and electric fields. If a particle is simply moving in an electric field, then you know that

$F = Eq$

where F is the force experienced by the particle, E is the strength of the magnetic field and q is the charge on the particle.

There are useful devices that use a combination of electric and magnetic fields. An example is a velocity selector. This device uses a combination of electric and magnetic fields to trap particles moving at different speeds. When the force on a particle as a result of the electric field is the same as the force on the particle as a result of the magnetic field

$F = Eq = qvB\sin\theta$

Hence $v = \frac{E}{B}\sin\theta$

Worked example 6.4

Find the speed of an electron travelling at 90° to the electric and magnetic fields in a velocity selector operating with an electric field of 3.0 kV and a magnetic field of 3.0 T.

v (m/s)	E (V)	B (T)	$\sin\theta$
?	3.0×10^3	3.0	1

Use $v = \frac{E}{B}\sin\theta$

$= \frac{3 \times 10^3}{3}$

$= 1 \times 10^3$ m/s

Activity 6.4: Researching how electric and magnetic fields are used in traditional television and computer screens

In a small group, carry out some research to find out how electric and magnetic fields are used in traditional television and computer screens.

J.J. Thompson's experiment of charge to mass ratio

J.J. Thompson used balanced electric and magnetic fields to measure the charge to mass ratio for an electron. His apparatus is shown in Figure 6.9.

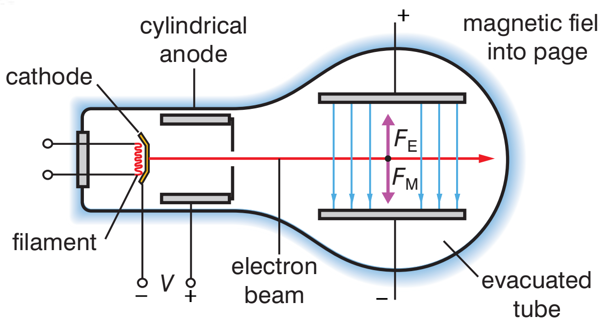

Figure 6.9 *Measuring the change to mass ratio of an election*

In this case $\theta = 90°$. We know that in such circumstances

$$v = \frac{E}{B}$$

We can find another expression for v by using the fact that the electron beam is accelerated by the potential difference between the cathode and the anode. We know that the kinetic energy of the electrons is given by

$$\frac{1}{2}mv^2 = qV$$

where v is the velocity of an electrons, m is the mass of an electron, q is the charge on an electron and V is the accelerating potential difference.

We can rearrange this equation to

$$v = \sqrt{\frac{2qV}{m}}$$

If we equate this to the expression for v involving E and B, and square both sides, we get

$$\frac{E^2}{B^2} = \frac{2qV}{m}$$

We can rearrange this to get

$$\frac{q}{m} = \frac{E^2}{2VB^2}$$

DID YOU KNOW?

John Joseph Thomson won the Nobel Prize for Physics in 1906. In 1937, his son, George Paget Thomson, won the same prize.

Worked example 6.5

a) Find the charge mass ratio for an electron accelerated through 600 V in a magnetic field of strength 45 mT where the speed of the electron is 1.4×10^7 m/s.

b) What is the percentage difference between your result and the accepted ratio with the values $q = 1.6 \times 10^{-19}$ C and $m = 9.11 \times 10^{-31}$ kg?

a)

v (m/s)	V (V)	q/m (C/kg)
1.4×10^7	600	?

Use

$$v = \sqrt{\frac{2qV}{m}}$$

Square both sides

$$v^2 = \frac{2qV}{m}$$

Rearrange

$$\frac{q}{m} = \frac{v^2}{2V}$$

$$= \frac{(1.4 \times 10^7)^2}{2 \times 600}$$

$= 1.63 \times 10^{11}$ C/kg

b) Ratio with accepted values $= \dfrac{1.6 \times 10^{-19}}{9.11 \times 10^{11}}$

$= 1.76 \times 10^{11}$

Percentage difference $= \dfrac{1.76 \times 10^{11} - 1.63 \times 10^{11}}{1.76 \times 10^{11}} \times 100$

$= 7.4\%$

Circular motion of particles in magnetic fields

From the right hand rule, you know that the force on a charged particle is always at right angles to the direction of its velocity. The force therefore acts as a centripetal force and so the particle follows a circular path.

You know that $F = qvB$ and $F = \dfrac{mv^2}{r}$

where q is the charge on the particle, v is the velocity of the particle, B is the strength of the magnetic field and r is the radius of the circular path.

From this we can see that

$$r = \frac{mv}{qB}$$

We can also find the period, T, since

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{qB}$$

The frequency, f, is the inverse of the period so

$$f = \frac{qB}{2\pi m}$$

The angular velocity, ω, is $2\pi f$, so

$$\omega = \frac{qB}{m}$$

The mass spectrometer

A mass spectrometer is a machine that allows chemicals to be separated according to their mass. A simplified diagram of a mass spectrometer is shown in Figure 6.10.

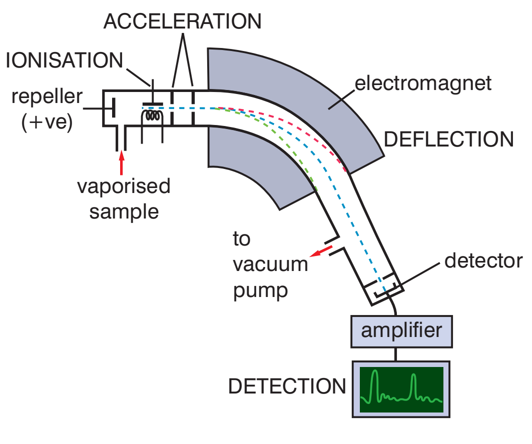

Figure 6.10 *A mass spectrometer*

The chemical enters the machine and is ionised (charged). It is then accelerated by an electric field and then its direction is changed when it enters a magnetic field.

In the last section, we learnt that in a magnetic field, a charged particle experiences a force as a result of the magnetic field,

$F = Bqv$

and a centripetal force

$F = \frac{mv^2}{r}$

Particles that follow the central dotted path in Figure 6.9 will reach the detector. For these particles

$F = Bqv = \frac{mv^2}{r}$ where r is the radius of the circular path.

We can rearrange this to $\frac{q}{m} = \frac{v}{Br}$

We can identify the particles by the value of the charge : mass ratio, and the values of B and r are known from the calibration of the machine. So we just need to know the speed of the particles when they entered the electromagnet. From the section on J. J. Thomson's experiment above, we know that

$$v = \sqrt{\frac{2qV}{m}}$$

If we substitute this into the equation for the charge to mass ratio

$$\frac{q}{m} = \frac{\sqrt{2qV}}{Br\sqrt{m}}$$

Square both sides

$$\frac{q^2}{m^2} = \frac{2qV}{B^2r^2m}$$

Thus

$$\frac{q}{m} = \frac{2V}{B^2r^2}$$

So we can find the mass of a particle using

$$m = \frac{B^2r^2q}{2V}$$

So by adjusting the accelerating voltage and the strength of the electromagnet (by changing the current through it) we can identify different chemicals in a sample.

Summary

In this section you have learnt that:

- A moving charge sets up a magnetic field.
- A charge moving in a magnetic field will travel in a circular path whose radius, r, is given by $r = \frac{mv}{qB}$ where m is the mass of the particle, v is the speed of the particle, q is the charge on the particle and B is the strength of the magnetic field.
- The equation $F = qv \times B$ is used to determine the magnitude and direction of the force.
- If $\theta \neq 90°$ then the force on the particle will be reduced by a factor sin θ from its maximum value, which occurs when the particle is travelling perpendicular to the magnetic field.
- J.J. Thompson's experiment to find charge to mass ratio is based on applying equal forces from an electric field and from a magnetic field to a charged particle. Two expressions for the velocity of the particle are found, one from the balanced forces and the other from the kinetic energy of the particle. These two expressions are equated and a value for the charge to mass ratio is found to be $\frac{q}{m} = \frac{E^2}{2VB^2}$

Review questions

1. Derive an expression for the radius, r, of the circular path of a particle of charge q and of mass m moving at speed v in a magnetic field of strength B.
2. Find the size of the force felt by an electron travelling at an angle of 50° to the Earth's magnetic field at 1.4×10^7 m/s. (The charge on an electron is 1.6×10^{-19} C and the magnitude of the Earth's magnetic field is 5×10^{-5} T.)
3. Isotopes of iron (Fe) are to be separated using a mass spectrometer. The applied magnetic field is 45 mT and the applied potential difference is 600 V. The mass of a proton or neutron is 1.66×10^{-27} kg and the charge on a proton is 1.6×10^{-19} C. Find the radii of the paths of ^{54}Fe, ^{56}Fe and ^{57}Fe.

6.4 Magnetic force on current-carrying conductors (long, straight, circular loop)

By the end of this section you should be able to:

- Derive the expression $F = I(l \times B)$.
- Use the expression for the force on a current-carrying conductor in a magnetic field.
- Determine the magnitude and direction of torque acting on a current loop.
- Define magnetic dipole moment.
- Describe the working mechanism of a direct motor.
- Describe and illustrate the magnetic field produced by an electric current in a long straight conductor and in a solenoid.
- Calculate the magnetic field strength of a straight current carrying wire.
- Analyse and predict using the right hand rule the direction of the magnetic field produced when electric current flows through a long straight conductor and a solenoid
- State the Biot–Savart law.
- Apply and use the Biot–Savart law to determine the expression for magnetic field strength of a current element.

The force on a current-carrying conductor in a magnetic field

We know that the force on a charge travelling in a magnetic field is given by

$F = Bqv\sin\theta$.

For a charge travelling a length l in the wire as shown in Figure 6.11, we can substitute $\frac{l}{t}$ for v so the equation becomes $F = Bq\frac{l}{t}\sin\theta$

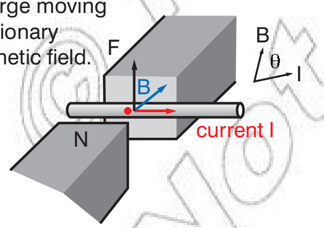

Figure 6.11

We also know that current, I, is $\frac{q}{t}$

So the equation becomes

$F = BIl\sin\theta$

The direction of the force is perpendicular to both the wire and the magnetic field and is given by the right hand rule as shown in Figure 6.12.

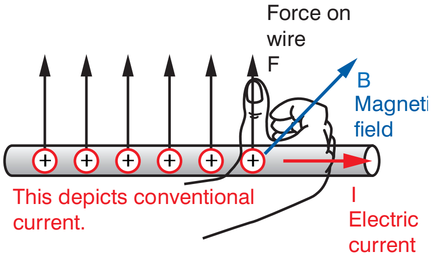

Figure 6.12 The direction is perpendicular to both wire and magnetic field.

We can show that the force is a vector (has both magnitude and direction) if we write the equation as follows $\vec{F} = \vec{I}(l \times B)$.

Activity 6.5: The variation of the magnetic field due to a current-carrying conductor

In a small group, investigate the variation of magnetic field due to a current-carrying conductor. Use a ring stand and a clamp to hold a piece of cardboard horizontally. Thread connecting wire through a hole in the cardboard, then connect the wire to a battery, a variable resistor (so that you can vary the current later) and a switch. Place several compasses on the cardboard around the wire.

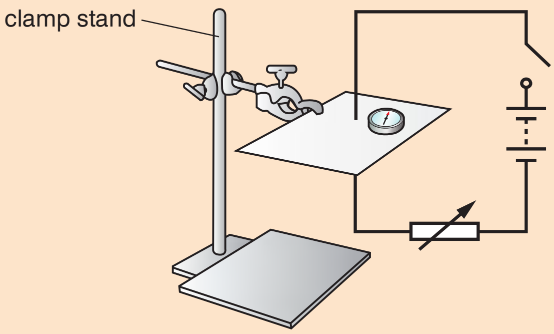

Figure 6.13

Make sure the wire is vertical and close the switch. Observe the behavior of the compasses and sketch the magnetic field lines near the current-carrying wire. Discuss:

- how you decided to draw the lines as you did
- how the field lines that you drew differ from those around a bar magnet.

Repeat this activity with a different current through the wire. How does the field pattern change?

Activity 6.6: The effect of the transverse force

Set up the apparatus as shown in Figure 6.14.

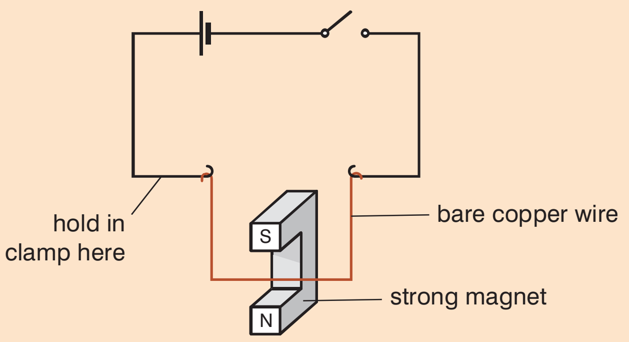

Figure 6.14

Allow a current of up to 5 A to flow through the wire. Observe what happens. Reverse the direction of the current. What happens now?

Worked example 6.6

A student sets up a jumping wire demonstration to impress her younger cousin. She uses a wire with a current of 1.5 A running through it, and a pair of magnets, which have a magnetic field of 0.75 mT. She is a bit careless in setting up and 5 cm of the wire hangs across the field at an angle of 75°.

a) How much force does the wire experience?

b) If it has a mass of 7.5 g, how fast will it accelerate initially?

a)

F (N)	I (A)	l (m)	B (T)	sin θ
?	1.5	0.05	0.75×10^{-3}	0.966

Use $F = BIl\sin\theta$

$= 0.75 \times 10^{-3} \times 1.5 \times 0.05 \times 0.966$

$= 5.44 \times 10^{-5}$ N

b)

F (N)	m (kg)	a (m/s²)
5.44×10^{-5} N	0.0075	?

Use $F = ma$

$$a = \frac{F}{m}$$

$$= \frac{5.44 \times 10^{-5}}{0.0075}$$

$= 7.25 \times 10^{-3}$ m/s²

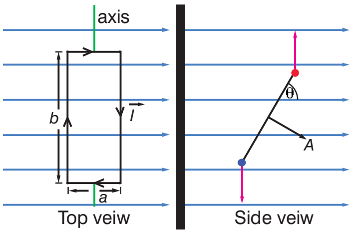

Figure 6.15

The magnitude and direction of torque acting on a current loop

Consider a rectangular loop of wire in a uniform magnetic field, as shown in Figure 6.15.

The sides with length a are parallel to the magnetic field and so they do not experience a force since sin θ = 0.

The sides with length b are perpendicular to the magnetic field and so each will experience a force of magnitude

$F = Bib$ (sin θ = 1 for these sides)

However, since the direction of the current is opposite on each side, the direction of the force will be opposite so there is no net force. However, there will be a resultant torque, τ. This torque acts about the axis and is given by

$$\tau = F\frac{a}{2} + F\frac{a}{2} = Fa = IbBa$$

The area of the loop, A, is ab, and so the torque is given by

$$\tau = IBA$$

This is the maximum torque, when the field is in the plane of the loop. In general the torque is given by

$$\tau = I(\vec{A}\times\vec{B})$$

which is also written as

$$\tau = IBA\sin\theta$$

where θ is the angle between the plane of the loop and the field (shown in the side view in Figure 6.15).

You can find the directions of the vectors in this equation using a right hand rule: curl the fingers of your right hand in the direction of the current and your thumb, stuck out, points in the direction of the area vector.

Worked example 6.7

Find the torque on a loop of wire of area 10 cm^2 at 60° to a magnetic field of strength 30 mT with a current of 2 A flowing through it.

τ (N m)	I (A)	B (T)	A (m^2)	sin θ
?	2	30×10^{-3}	10×10^{-4}	0.866

Use $\tau = IBA\sin\theta$

$= 2 \times 30 \times 10^{-3} \times 10 \times 10^{-4} \times 0.866$

$= 5.196 \times 10^{-5}$ N m

Magnetic dipole moment

A current loop creates a magnetic dipole moment. A dipole moment is defined as

current (I) × area (A)

If we consider a coil of wire consisting of N loops, then the magnetic moment, μ, of such a coil is given by

$\mu = NIA$

The direction of the magnetic moment is given by the right hand rule.

When a magnetic dipole moment is placed in a magnetic field (B), it experiences a torque. This torque is given by the equation

$\tau = \mu \times B$

This can be written as

$\tau = \mu B \sin \theta$

Worked example 6.8

a) Find the magnetic dipole moment on a coil of wire with 100 turns each of area 10 cm² at 60° to a magnetic field of strength 30 mT with a current of 2 A flowing through it.

b) Find the torque on the coil described in part (a).

a)

N	I (A)	A (m²)	μ (A m²)
100	2	10×10^{-4}	?

Use μ = NIA

$= 100 \times 2 \times 10 \times 10^{-4}$

$= 0.2$ A m²

b)

τ (N m)	μ (A m²)	B (T)	sin θ
?	0.2	30×10^{-3}	0.866

Use $\tau = \mu B \sin \theta$

$= 0.2 \times 30 \times 10^{-3} \times 0.866$

$= 5.196 \times 10^{-3}$ N m

The working mechanism of a direct current motor

The principle of the direct current motor is the forced movement of a current carrying loop in a magnetic field. The torque on a current carrying loop will cause it to rotate. If it is free to move then the loop will rotate continuously. You can find the direction of the force using a right hand rule again: the thumb points in the direction of conventional current, the index finger in the direction of the magnetic field and the middle finger in the direction of the force. A direct current motor will have a coil with many turns of wire but Figure 6.16 simplifies the situation and just shows a single rectangular loop.

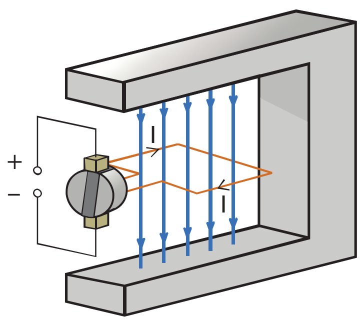

Figure 6.16 *A direct current motor*

DID YOU KNOW?

In 2003, the Zettl Lab at the University of Berkeley in California produced a motor which is was less than 500 nm across.

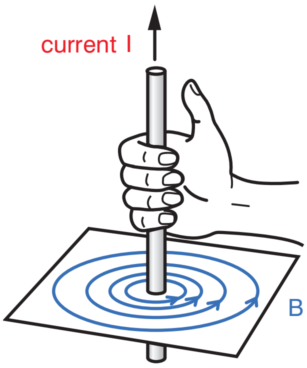

***Figure 6.17** Magnetic field produced by current in a straight conductor*

The magnetic field produced by an electric current in along straight conductor

In Activity 6.5 on page 248, you should have found that the magnetic field around a long straight wire takes the form of concentric circles around the wire. You find the direction of the magnetic field by wrapping the fingers of your right hand around the wire with your thumb in the direction of the current, as shown in Figure 6.17.

The strength of the magnetic field, B, depends on

- the current, I, flowing through the conductor
- the inverse of the distance from the conductor, r (in other words, as the distance from the conductor increases, the strength of the field decreases).

Mathematically, we can write this as

$$B = \frac{kI}{r}$$

where k is a constant term.

It has been found that the value of k depends on the value known as the permeability of free space, μ_0, and the inverse of 2π. So

$$k = \frac{\mu_0}{2\pi}$$

We can see that, for a straight current-carrying conductor

$$B = \frac{\mu_0 I}{2\pi r}$$

Worked example 6.9

In Activity 6.5, you varied the current through your current-carrying wire and plotted the magnetic field. Calculate what the strength of the magnetic field would have been 10 cm from the wire in your experiment if your wire had been carrying a current of 2 A. The permeability of free space is $4\pi \times 10^{-7}$ T m/A

B (T)	μ_0 (T m/A)	I (A)	r (m)
?	$4\pi \times 10^{-7}$	2	0.1

Use

$$B = \frac{\mu_0 I}{2\pi r}$$

$$= \frac{4\pi \times 10^{-7} \times 2}{2\pi \times 0.1}$$

$$= 4 \times 10^{-6} \text{ T}$$

The magnetic force between two wires

If two identical parallel wires each carry current, as shown in Figure 6.18, then each will exert a force F on the other.

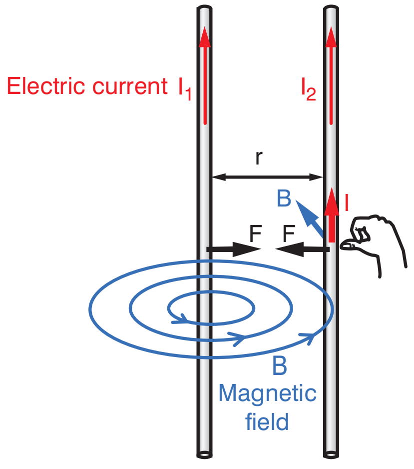

***Figure 6.18** Magnetic force between two wires*

The magnetic field in wire 2 from I_1 is given by

$$B = \frac{\mu_0 I_1}{2\pi r}$$

The force on length Δl of wire 2 is given by

$$F = I_2 \Delta l B$$

The force per unit length in terms of the currents is therefore

$$\frac{F}{\Delta l} = \frac{I_2 \mu_0 I_1}{2\pi r}$$

We can rearrange this to give

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

Biot–Savart law and determining the expression for magnetic field strength of a current element

The Biot–Savart law relates magnetic fields to the currents which are their sources (in the same way as Coulomb's law relates electric fields to the point charges which are their sources).

Consider a current-carrying conductor as shown in Figure 6.19.

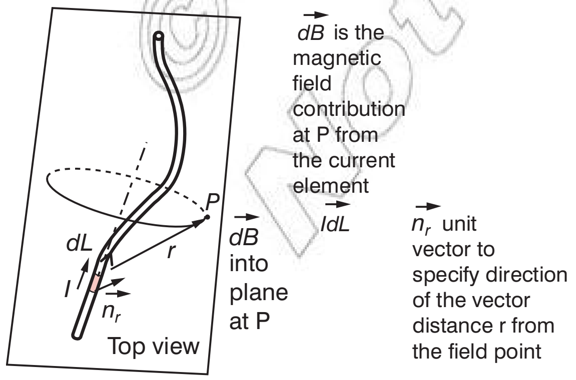

Figure 6.19

Each infinitesimal current element I (shown shaded in Figure 6.19) makes a contribution $d\vec{B}$ to the magnetic field at point P. P is perpendicular to the current element and perpendicular to the radius vector r from the current element.

$$d\vec{B} = \frac{\mu_0 I d\vec{l} \times \vec{n}_r}{4\pi r^2}$$

Summary

In this section you have learnt that:

- The expression $F = I(l \times B)$ is used to find the force on a current–carrying conductor in a magnetic field.
- The magnitude of a torque acting on a current loop is given by

  $\tau = IAB\sin\theta$

  where I is current through the loop, A is area of loop, B is magnetic field strength and θ is angle between magnetic field and loop
- A current loop creates a magnetic dipole moment. A dipole moment is defined as:
  current (I) $\times$ area (A)
  For a coil of wire consisting of N loops has a magnetic moment, μ, given by $\mu = NIA$
- The direction of the magnetic moment is given by the right hand rule.
- When a magnetic dipole moment is placed in a magnetic field (B), it experiences a torque. This torque is given by the equation:
  $\tau = \mu \times B$
  This can be written as:
  $\tau = \mu B\sin\theta$
- The principle of the direct current motor is the forced movement of a current carrying loop in a magnetic field. The torque on a current carrying loop will cause it to rotate. If it is free to move then the loop will rotate continuously. You can find the direction of the force using a right hand rule again: the thumb points in the direction of conventional current, the index finger in the direction of the magnetic field and the middle finger in the direction of the force.
- The magnetic field produced by an electric current in along straight conductor is concentric circles. It can be calculated using the equation $B = \frac{\mu_0 I}{2\pi r}$.

- Its direction is given using the right hand rule.
- The Biot–Savart law can be used to determine the expression for magnetic field strength of a current element $d\vec{B} = \frac{\mu_0 I d\vec{l} \times \vec{n}_r}{4\pi r^2}$.

Review questions

1. a) How much force does a wire with a current of 1.25 A running through it when it is set in magnetic field of 0.5 mT if 7.5 cm of the wire hangs across the field at an angle of 50°?

   b) If the wire has a mass of 10 g, how fast will it accelerate initially?
2. Find the torque on a loop of wire of area 8 cm^2 at 45° to a magnetic field of strength 60 mT with a current of 4.5 A flowing through it.
3. a) Find the magnetic dipole moment on a coil of wire with 500 turns each of area 5 cm^2 at 75° to a magnetic field of strength 25 mT with a current of 1.5 A flowing through it.

   b) Find the torque on the coil described in part a).
4. Describe the principle of the direct current motor.
5. Calculate the strength of the magnetic field 15 cm from a straight current-carrying wire if the wire carries a current of 1 A. The permeability of free space is $4\pi \times 10^{-7}$ T m/A.
6. Calculate the force between two parallel wires each of 1 m in length that are 1 m apart when each carries a current of 1 A. The permeability of free space is $4\pi \times 10^{-7}$ T m/A.
7. Find the contribution to the magnetic field at a point that is a perpendicular distance 10 cm from a current element of length 10 cm, where the current through the element is 1.5 A. The permeability of free space is $4\pi \times 10^{-7}$ T m/A.

6.5 Ampere's law and its application

By the end of this section you should be able to:

- State Ampere's law and use it in solving problems.
- Describe and illustrate the magnetic field produced in a solenoid and predict its direction using the right hand rule.

DID YOU KNOW?

Origin of Ampere's law

In 1820, Oersted discovered that electric currents can induce magnetic fields. Several weeks later Ampere went to a talk in Paris where Oersted's discovery was reported. Ampere began to do detailed experiments to investigate the nature of these induced magnetic fields and their relationship to the electric currents. His research is summed up in Ampere's law.

Ampere's law

Ampere's law states that in any closed loop path, the sum of the length elements times the magnetic field in the direction of the length element is equal to the permeability of free space times the electric current enclosed in the loop, as shown in Figure 6.20.

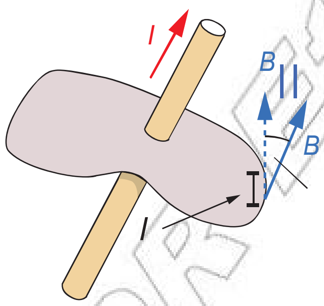

Figure 6.20

Mathematically, we write this as

$$\sum B_{||}\Delta l = \mu_0 I$$

If the angle between the parallel component and the path is θ then the expression becomes

$$\sum B\Delta l\cos\theta = \mu_0 I$$

Ampere's law and the magnetic field inside a wire

We have already found the expression for the magnetic field outside a current-carrying conductor. We can confirm this expression using Ampere's law since, in this case, $\Delta l = 2\pi r$ (the expression for the circumference of a circle). Hence, using

$$\sum B_{||}\Delta l = \mu_0 I$$

and substituting $\Delta l = 2\pi r$, we get

$$B = \frac{\mu_0 I}{2\pi r}$$

We can use Ampere's law to derive an expression for the magnetic field inside a current-carrying conductor. Consider the conductor shown in Figure 6.21.

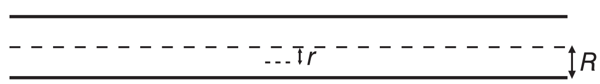

Figure 6.21

At a distance r from the centre of a conductor of radius R, the current enclosed is given by

$$\frac{Ir^2}{R^2}$$

Using Ampere's law we get

$$\sum B \times 2\pi r = \mu_0 \frac{Ir^2}{R^2}$$

which simplifies to

$$B = \frac{\mu_0 Ir}{2\pi R^2}$$

Worked example 6.10

Find the magnetic field strength inside a conductor of radius 10 mm at a distance of 3 mm from the centre of the conductor when a current of 1.5 A is flowing through the conductor. The permeability of free space is $4\pi \times 10^{-7}$ T m/A.

B (T)	μ_0 (T m/A)	I (A)	r (m)	R (m)
?	$4\pi \times 10^{-7}$	1.5	3×10^{-3}	10×10^{-3}

Use

$$B = \frac{\mu_0 Ir}{2\pi R^2}$$

$$= \frac{4\pi \times 10^{-7} \times 1.5 \times 3 \times 10^{-3}}{2\pi \times 10 \times 10^{-3} \times 10 \times 10^{-3}}$$

$$= \frac{9 \times 10^{-10}}{1 \times 10^{-4}}$$

$$= 9 \times 10^{-6} \text{ T}$$

Ampere's law and the magnetic field of a solenoid

A long straight coil of wire, called a solenoid, can be used to generate a magnetic field that is similar to that of a bar magnet. Such coils have many practical applications. The field can be strengthened by adding an iron core. Such cores are typically used in electromagnets.

You can use Ampere's law to find the magnetic field B for a solenoid. Consider the solenoid shown in Figure 6.22.

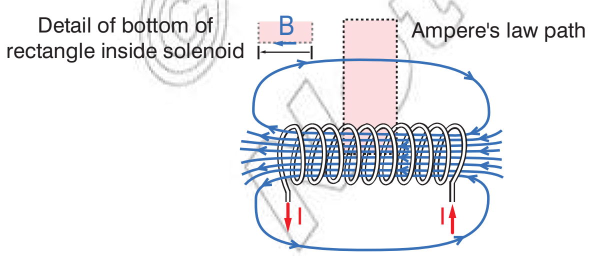

Figure 6.22

If we take a rectangular path so that the length of the side parallel to the solenoid field is length l (shown shaded in Figure 6.22), the contribution to the field from this path is Bl inside the coil where

B is the magnetic field strength. The field can be considered to be perpendicular to the sides of the path so these give negligible contribution.

Using Ampere's law we get, for a solenoid of N turns,

$$Bl = \mu_0 NI$$
$$B = \frac{\mu_0 NI}{l}$$
$$B = \mu_0 nI$$

where n is number of turns per unit length.

We can find the direction of the magnetic field in a solenoid using the right hand rule, as for a current-carrying wire.

Activity 6.7: Making a solenoid

Wrap a 20 cm piece of insulated wire around a pencil several times. Remove the pencil and place the coil of wire on a piece of cardboard. Connect the coil to a battery through two holes in the cardboard. Place several compasses around the coil. Sketch the coil and the compasses in your notebook, then close the switch. Observe the orientation of the compass needles.

Change the shape of the wire and repeat. Compare the shape of the magnetic field for different shapes of wire.

Worked example 6.11

Find the magnetic field inside a solenoid of 1000 turns per unit length with a current of 3 A flowing through it. The permeability of free space is $4\pi \times 10^{-7}$ T m/A.

B (T)	μ_0 (T m/A)	n	I (A)
?	$4\pi \times 10^{-7}$	1000	3

Use $B = \mu_0 nI$

$= 4\pi \times 10^{-7} \times 1000 \times 3$

$= 3.768 \times 10^{-3}$ T

Activity 6.8: Investigating the force of attraction between a solenoid and a bar magnet for different values of current through the solenoid

Work in a small group to design and carry out an investigation into the force of attraction between a bar magnet and a solenoid with varying current through the solenoid. What apparatus will you need? What measurements will you need to take? How will you display your results? Write a report on this investigation. Remember that your report should be sufficiently detailed to enable the reader to repeat your procedure and check your results!

Ampere's law and the magnetic field of a toroid

Consider a toroid, as shown in Figure 6.23.

All the loops that make up the toroid contribute magnetic field in the same direction inside the toroid. The direction of the magnetic field can be found using the right hand rule (compare with a solenoid). The current enclosed by the dashed line is just

NI

where N is the number of loops and I is the current in each loop.

Ampere's law then gives the magnetic field since

$B \times 2\pi r = \mu_0 NI$

So

$$B = \frac{\mu_0 NI}{2\pi r}$$

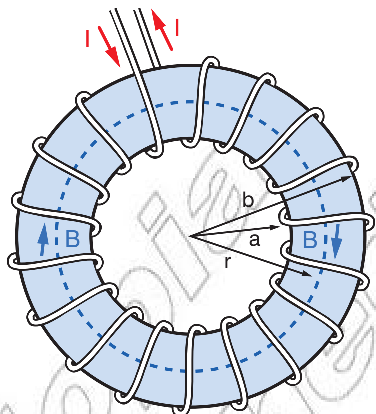

Figure 6.23 *A toroid*

Worked example 6.12

Find the magnetic field for a toroid of radius 5 cm with 1000 turns per unit length with a current of 3 A flowing through it. The permeability of free space is $4\pi \times 10^{-7}$ T m/A.

B (T)	μ_0 (T m/A)	n	I (A)	r (m)
?	$4\pi \times 10^{-7}$	1000	3	0.05

Use

$$B = \frac{\mu_0 NI}{2\pi r}$$

$$= \frac{4\pi \times 10^{-7} \times 1000 \times 3}{2\pi \times 0.05}$$

$= 0.012$ T

Summary

In this section you have learnt that:

- Ampere's law states that in any closed loop path, the sum of the length elements times the magnetic field in the direction of the length element is equal to the permeability of free space times the electric current enclosed in the loop, as shown in Figure 6.24.

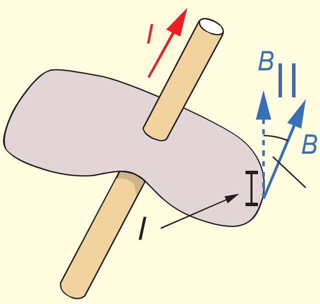

Figure 6.24

Mathematically, we write this as

$$\sum B_{||} \Delta l = \mu_0 I$$

If the angle between the parallel component and the path is θ then the expression becomes

$$\sum B \Delta l \cos\theta = \mu_0 I$$

- Ampere's law can be used to derive the magnetic field strength for a solenoid $B = \mu_0 n I$ where n is number of turns per unit length, the magnetic field inside a conductor

$$B = \frac{\mu_0 I r}{2\pi R^2}$$

and the magnetic field from a toroid

$$B = \frac{\mu_0 N I}{2\pi r}$$

- The magnetic field produced in a solenoid is similar to that of a bar magnet and its direction can be predicted using the right hand rule.

Review questions

1. State Ampere's law.
2. Find the magnetic field strength inside a conductor of radius 20 mm at a distance of 5 mm from the centre of the conductor when a current of 3 A is flowing through the conductor. The permeability of free space is $4\pi \times 10^{-7}$ T m/A.
3. Find the magnetic field inside a solenoid of 500 turns per unit length with a current of 1.5 A flowing through it. The permeability of free space is $4\pi \times 10^{-7}$ T m/A.
4. Find the magnetic field for a toroid of radius 3 cm with 500 turns per unit length with a current of 2 A flowing through it. The permeability of free space is $4\pi \times 10^{-7}$ T m/A.

6.6 Earth's magnetism

By the end of this section you should be able to:

- Determine the horizontal component of the Earth's magnetic field at a location.
- Resolve the horizontal and vertical components of the Earth's magnetic field.
- Describe how a tangent galvanometer works.

Horizontal and vertical components of the Earth's magnetic field

Except near the equator, the field lines of the Earth's magnetic field are at an angle to the Earth's surface. At the magnetic poles, the field lines pass through the Earth's surface vertically. However, at any other point on the Earth's surface the Earth's magnetic field has a vertical and a horizontal component.

The Earth's magnetic field is a vector quantity; at each point in space it has a strength and a direction.

The strength of the field at the Earth's surface ranges from less than 30 000 nT America and South Africa to over 60 000 nT around the magnetic poles in northern Canada and south of Australia, and in part of Siberia. Near the poles, the field strength diminishes with the inverse square of the distance, i.e. at a distance of R Earth radii it only amounts to $1/R^2$ of the surface field in the same direction, whereas at greater distances, such as in outer space, it diminishes with the cube of the distance. Where the prime meridian intersects with the equator, the field strength is about 31 μT.

DID YOU KNOW?

Generally, the Earth's magnetic field strength is equivalent to 1/30 000th of a tesla. Still, this is enough for birds to navigate by and to keep a compass hand pointed north. The magnetic field of Jupiter, the largest planet in the solar system, is about ten times stronger than Earth's, or 1/3000th of a tesla.

The tangent galvanometer

The tangent galvanometer (TG) is an instrument for measuring the strength of an electrical current in terms of the magnetic field it produces. A TG consists of a circular coil of insulated copper wire wound on a circular non magnetic frame. The frame is mounted vertically on a horizontal base provided with levelling screws on the base. The coil can be rotated on a vertical axis passing through its centre. A compass box is mounted horizontally at the centre of a circular scale. The compass box is circular in shape. It consists of a tiny, powerful magnetic needle pivoted at the centre of the coil. The magnetic needle is free to rotate in the horizontal plane. The circular scale is divided into four quadrants. Each quadrant is graduated from 0° to 90°. A long thin aluminium pointer is attached to the needle at its centre and at right angle to it. To avoid errors due to parallax a plane mirror is mounted below the compass needle.

Figure 6.25 *A tangent galvanometer*

Current flowing through a coil of wire generates a magnetic field at the centre of a coil, and this field deflects a magnetic compass needle. The instrument derives its name from the fact that the current is proportional to the tangent of the angle of the needle's deflection. When current is passed through the TG a magnetic field is created at its centre. This field is given by

$$B_c = \frac{\mu_0 N I}{2R}$$

where N is the number of turns of wire in the coil, I is the current through it and R is the radius of the coil.

DID YOU KNOW?

The Magnetic Observatory of Addis Ababa has been operating since January 1958. In August 1997, the Institut de Physique du Globe de Paris (IPGP) installed a new magnetic station in Addis Ababa as part of its network 'Observatoire Magnetique Planetaire'. In August 2004, a new vector magnetometer VM391 was installed. Its purpose is to provide ground-based, calibrated values of the Earth's magnetic field at a specific location for scientific research and practical applications.

If the TG is set such that the plane of the coil is along the magnetic meridian, i.e. B_c is perpendicular to the horizontal component of the Earth's magnetic field, the needle rests along the resultant, as shown in Figure 6.26.

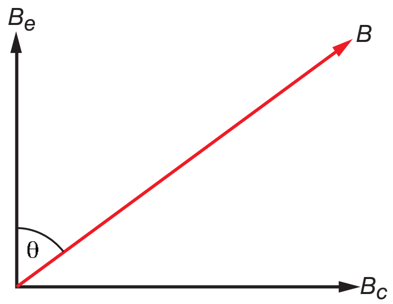

Figure 6.26

Because a compass aligns itself with the lines of force of the magnetic field within which it is placed, a compass can be used to find the angle θ between B_e and B. If the compass is first aligned with the magnetic field of the Earth and current is supplied to the coils, then the compass needle will undergo an angular deflection aligning itself with the vector sum of the Earth's field and the field due to the coils. This angular deflection is θ.

The horizontal component of the Earth's magnetic field can be expressed as

$$B_e = \frac{B_c}{\tan \theta}$$

Activity 6.9: Using a tangent galvanometer to determine the strength of the Earth's magnetic field at your location

If possible, use a tangent galvanometer to determine the strength of the Earth's magnetic field at your location. If this is not possible, carry out some research into the Magnetic Observatory at Addis Ababa.

Summary

In this section you have learnt that:

- The Earth's magnetic field is a vector quantity; at each point in space it has a strength and a direction.
- Except near the equator, the field lines of the Earth's magnetic field are at an angle to the Earth's surface. At the magnetic poles, the field lines pass through the Earth's surface vertically. However, at any other point on the Earth's surface the Earth's magnetic field has a vertical and a horizontal component.
- The strength of the field at the Earth's surface ranges from less than 30 000 nT in America and South Africa to over 60 000 nT around the magnetic poles in northern Canada and south of Australia, and in parts of Siberia.
- The tangent galvanometer (TG) is an instrument for measuring the strength of an electrical current in terms of the magnetic field it produces.
- The current is proportional to the tangent of the angle of the needle's deflection. When current is passed through the TG a magnetic field is created at its centre. This field is given by

  $$B_c = \frac{\mu_0 NI}{2R}$$

  where N is the number of turns of wire in the coil, I is the current through it and R is the radius of the coil.
- If the TG is set such that the plane of the coil is along the magnetic meridian, i.e. B_c is perpendicular to the horizontal component of the Earth's magnetic field, the needle rests along the resultant, as shown in Figure 6.27.

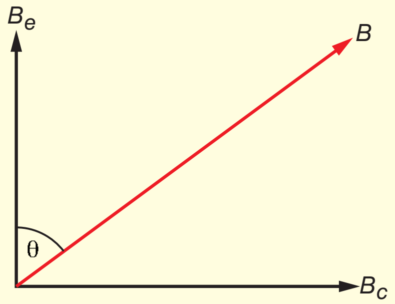

Figure 6.27

- The horizontal component of the Earth's magnetic field can be expressed as

  $$B_e = \frac{B_c}{\tan \theta}$$

Review questions

1. What instrument could be used to determine the horizontal component of the Earth's magnetic field?
2. At a particular location, $\theta = 70°$. If coils of radii 10 cm with 1000 turns and a current of 5 A were used to determine the horizontal component of the Earth's magnetic field at this location, what measurement would be given for the strength of this component at this location? The permeability of free space is $4\pi \times 10^{-7}$ T m/A.

End of unit questions

1. Describe an experiment to demonstrate the three-dimensional nature of the magnetic field around a bar magnet.
2. A bar magnet causes a magnetic field with a strength of 30 mT at an angle of 45° to a region of area 15 cm^2. How much flux will be contained by this region?
3. What does the term 'magnetism' describe?
4. Explain the difference between paramagnetic and ferromagnetic materials.
5. Outline the dynamo theory which can be used to explain the Earth's magnetic field.
6. Find the speed of an electron travelling at an angle of 40° to the Earth's magnetic field that experiences a force of 8×10^{-17} N. (The charge on an electron is 1.6×10^{-19} C and the magnitude of the Earth's magnetic field is 5×10^{-5} T.)
7. Outline J.J. Thompson's experiment of charge to mass ratio.
8. Outline the principles of a mass specrometer.
9. Isotopes of carbon (C) are to be separated using a mass spectrometer. The applied magnetic field is 45 mT and the applied potential difference is 600 V. The mass of a proton or neutron is 1.66×10^{-27} kg and the charge on a proton is 1.6×10^{-19} C. Find the radii of the paths of ^{12}C, ^{13}C and ^{14}C.
10. Describe how you would investigate the variation of the magnetic field due to a current-carrying conductor.

11 a) A straight piece of conducting wire, 20 cm long, lies at 70° a magnetic field of 1.8×10^{-5} T. A current, *I*, is allowed to flow through it and it experiences a force of 0.01 N. Calculate the value of *I*.

b) The wire in part (a) is bent into a rectangular shape so that it has an area of 24 cm^2. Find the torque on the loop of wire 70° to the magnetic field of strength 1.8×10^{-5} T when the same current as in part (a) flows through it.

c) Find the magnetic dipole moment on a coil of wire with 1000 turns each of area 24 cm^2 at 70° to a magnetic field of strength 1.8×10^{-5} T with the same current as part (a) flowing through the coil.

12. Describe the principle of the direct current motor
13. How far from a straight current–carrying wire carrying a current of 1 A is the strength of the magnetic field 1.5×10^{-6} T? The permeability of free space is $4\pi \times 10^{-7}$ T m/A.
14. Show how to derive the expression for the magnetic field between two wires.
15. Find the contribution to the magnetic field at a point that is a perpendicular distance 15 cm from a current element of length 20 cm, where the current through the element is 5 A. The permeability of free space is $4\pi \times 10^{-7}$ T m/A.
16. State Ampere's law.
17. How far from the centre of a conductor of radius 10 mm with a current of 1.5 A flowing through it is the magnetic field strength 5×10^{-6} T? The permeability of free space is $4\pi \times 10^{-7}$ T m/A.
18. Consider the solenoid shown in Figure 6.28

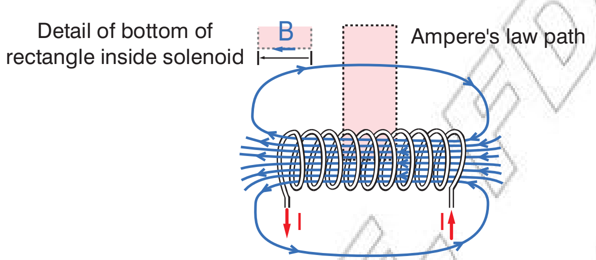

Figure 6.28

Derive the expression for the magnetic field of this solenoid.

19. What current is needed through a solenoid of 350 turns if the magnetic field inside it is to be 9×10^{-4}? The permeability of free space is $4\pi \times 10^{-7}$ T m/A.
20. Find the magnetic field for a toroid of radius 3 cm with 200 turns per unit length with a current of 2 A flowing through it. The permeability of free space is $4\pi \times 10^{-7}$ T m/A.
21. What is the range of the strength of the magnetic field at the earth's surface.
22. Describe how a tangent galvanometer works.